Vietnam National University **High School Entrance Exam 2017**

College of Science **Hanoi High School for Gifted Student**

Subject: Math - round II

Time limit: 150 minutes

Q1 (3.5pts) 1) Solve system of equation

$$\begin{cases} x+y=\sqrt{x+3y} \\ x^2+y^2+xy=3 \end{cases}$$

2) For $a,b \in \mathbb{R}^+$ s.t. $ab+a+b=1$, prove

$$\frac{a}{1+a^2}+\frac{b}{1+b^2}=\frac{1+ab}{\sqrt{2(1+a^2)(1+b^2)}}$$

Q2 (2.5pts) 1) For p,q be prime numbers s.t.

$$p(p-1)=q(q^2-1) \qquad (*)$$

- Prove that there exists a prime number $k \in \mathbb{N}^+$ that satisfies $p-1=kq, q^2-1=kp$
- Find all p,q that satisfy $(*)$

2) For $a,b,c \in \mathbb{R}^+$ s.t $ab+bc+ca+abc=2$, find the max of

$$M=\frac{a+1}{a^2+2a+2}+\frac{b+1}{b^2+2b+2}+\frac{c+1}{c^2+2c+2}$$

Q3 (3pts) Let ABC be an acute triangle with $AB<AC$ (length). E,F are midpoints of AC,AB respectively. The midperpendicular of EF crosses BC at D. Suppose there exists P inside $\widehat{EAF}$ (notation for angle, i.e., $\angle EAF$) and outside of the triangle AEF s.t. $\widehat{PEC}=\widehat{DEF}$ and $\widehat{PFB}=\widehat{DFE}$. PA crosses the circumscribed circle of PEF at $Q \neq P$

1) Prove $\widehat{EQF}=\widehat{BAC}+\widehat{DEF}$
2) The tangent at P of the circumscribed circle of PEF crosses CA,AB at M,N respectively. Prove that four points C,M,B,N lie on the same circle. Call this circle (K)
3) Prove that (K) is tangent to the circumscribed circle of AEF

Q4 (1pt) Let $n \in \mathbb{N}^+$, $n \geq 5$. Consider a convex polygon with n edges. We want to draw several diagonals so that they divide the convex polygon into k regions, each being a convex pentagon (any two regions do not overlap except for possibly the edges)

1) Prove that we can do this for $n=2018$, $k=672$
2) For $n=2017$, $k=672$, can we do this? Explain.