Sergey Markelov, December 27, 2016

Another solution of Problem 16 on pp. 54, 55 from Sergey Markelov, sergey.markelov@gmail.com

First, note that

$$x^4 - 14x^3 + 66x^2 - 115x + 66.25 = \left(x^2 - 7x + 8\right)^2 + \left(x - \frac{3}{2}\right)^2.$$

This representation implies that the equation

$$x^4 - 14x^3 + 66x^2 - 115x + 66.25 = 0$$

has no real solutions.

Second, if we try to factorize

$$x^4 - 14x^3 + 66x^2 - 115x + 66.25 = a \cdot b,$$

we arrive at

$$a = x^2 + \left[2\sqrt{2 + \sqrt{5}} - 7\right]x + \frac{25}{2} + 2\sqrt{5} - \sqrt{110 + 50\sqrt{5}},$$

and

$$a = x^2 - \left[2\sqrt{2 + \sqrt{5}} + 7\right]x + \frac{25}{2} + 2\sqrt{5} + \sqrt{110 + 50\sqrt{5}},$$

from which the four solutions on p. 55 follow immediately. Everything is elementary, but the solution is quite tricky.